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ANOVA and power analysis

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This document is an extract from a course on power analysis provided by Highland Statistics Ltd. Due to copyright we cannot provide the full notes online. If you use Brodgar for power analysis, we strongly advise that you buy: Zar, J.H. (1999). Biostatistical Analysis. Fourth Edition. The windows in the power analysis menu in Brodgar have exactly the same names as the relevant power analysis sections in Zar (1999). The text below explains some of the options, and each section number in Zar (1999) is given.

During 2006, we will make a book submission on this material.

1. The population mean

The theory of power analysis for the population mean is described in Section 7.5 of Zar (1999). The formulae used here are identical.

Brodgar options for sample size and estimation of the population mean

The power analysis functions in Brodgar are the only tools in the package that do not re-quire the user to import data. The window in Figure 1 shows the options for the sample size estimation for the population mean. One can either calculate the required sample size or the half-width of the confidence interval d. The choice can be made in the "Parameter to be found" listbox. If the half-width of the confidence interval is required, then change the "Pa-rameter to be found" listbox from "Sample size" to "Confidence interval half-width", and specify n. The formulae were taken from Chapter 7 in Zar (1999), and are given in Table 1.

Table 1. Formula used for the power analysis for sample size of the population mean.

	Equation	Equation in Zar (1999)
Sample size	$n = \dfrac{S^2 t^2_{a(2)n}}{d^2}$	7.7
Half width	$d = \sqrt{\dfrac{S^2 t^2_{a(2)n}}{n}}$	7.7 rewritten

The options in Brodgar are described in more detail below.

Parameter to be found:
- Sample size: Estimate n as a function of S^2, a and the half-width confidence interval d.
- Confidence interval half-width: estimate d as a function of S^2, n and a.

Produce scatterplot for…
- One of the factors (either n or d) can be varied according to a specified range. The ef-fect on the estimate n or d is visualised using a scatterplot.

Figure 1. Power analysis options for the population mean.

2. Power and sample size in tests concerning the mean

The theory of power and sample size in tests concerning the mean can be found in Section 7.6 of Zar (1999). The formulae used in this manual are identical.

Brodgar options for power and sample size in tests concerning the mean

The options for the power and sample size in tests concerning the mean are similar to those in Figure 1, and are discussed next. The formulae are given in Zar (1999) and are reproduced in Table 2.

Table 2. Formula used for the power and sample size in tests concerning the mean.

	Equation	Equation in Zar (1999)
Sample size	$n = \dfrac{S^2(t_{a,v} + t_{b(1),v})^2}{d^2}$	7.8
Minimum detectable difference	$d = \sqrt{\dfrac{S^2(t_{a,v} + t_{b(1),v})^2}{n}}$	7.9
Power of the test	$t_{b(1),v} = \dfrac{d}{\sqrt{\dfrac{S^2}{n}}} - t_{a,v}$	7.10

The Brodgar options are as follows. Change the main option in Figure 1 to "Power and sample size in tests concerning the mean".

Parameter to be found:
- Sample size.
- Power of the test.
- Detectable difference.

In all options, the test can be one or two tailed, depending on the null hypothesis. This will influence a and the associated t-value.

Produce scatterplot for...
- One of the factors in the equation can be varied according to a specified range. The effect on the estimate n, detectable difference, significance level or power is visualised using a scatterplot.

3. Power analysis for comparing means using parametric tests for unpaired data.

The theory of power analysis for sample size and estimation of the differences between two population means is described in Chapter 8 of Zar (1999). The formulae used in this manual are identical. We are still in the Brodgar window labelled 'Population mean tests'. But we are now in Section 8.3 in Zar (1999).

Brodgar options for sample size and estimation of the differences between two population means

Table 3. Formula used for the power and sample size in tests concerning the mean.

	Equation	Equation in Zar (1999)
Sample size	$n = \dfrac{2S_p^2 t_{\boldsymbol{a}(2),2(n-1)}^2}{d^2}$	8.20
Half-width confidence interval d	$d = \sqrt{\dfrac{2S_p^2 t_{\boldsymbol{a}(2),2(n-1)}^2}{n}}$	Rewrite 8.20

The Brodgar options are as follows. Change the main option in Figure 1 to "sample size and estimation of the differences between two population means". The options are as follows.

Parameter to be found:
- Sample size n.
- Confidence interval half-width d.

Produce scatterplot for…
- One of the factors in the equation can be varied according to a specified range.

Power and sample size in tests for differences between two means

We now include the power in the formulae. In Section 8.4 in Zar (1999), power analysis was used to address the following questions
1. What is the required sample size? How large should the sample be such that we achieve a certain precision?
2. What is the minimum detectable difference?
3. What is the power of the test?

The Brodgar options are given below.

Brodgar options for power and sample size for differences between two means

Change the main option in Figure 1 to "Power and sample size for differences between two means". The underlying formulae are given in Table 4.

Table 4. Formula used for the power and sample size for differences between two means.

	Equation	Equation in Zar (1999)
Sample size	$n \geq \dfrac{2S_p^2}{d^2}(t_{a,v} + t_{b(1),v})^2$	8.22
Minimum detectable difference	$d \geq \sqrt{\dfrac{2S_p^2}{n}}(t_{a,v} + t_{b(1),v})$	8.23
Power of the test	$t_{b(1),v} = \dfrac{d}{\sqrt{\dfrac{2S_p^2}{n}}} - t_{a,v}$	8.24

The options are as follows.

Parameter to be found:
- Sample size. This is equation (4.3).
- Minimum detectable difference. This is equation (4.4).
- Power of the test. This is equation (4.5).

Produce scatterplot for…
- One of the factors in the equation can be varied according to a specified range.

Paired data

This is Section 9.3 in Zar (1999). Use the same formulae as above.

4. Power and sample size for ANOVA

In Brodgar, we now click on the 'ANOVA Power tests'. The main options are:
1.   Power estimation from population means.
2.   Power, sample size and minimum detectable difference.
3.   Power estimation from ANOVA results

The theory can be found in Section 10.3 of Zar (1999).

In a power analysis that tests for fixed-effects, there are three related quantities.

1. The power of the test. This is the probability that if such an effect exists, we find it. It needs to calculate a quantity called (f) (lower case Greek phi). The underlying formula for the power is given by:

$$f = \sqrt{\frac{n \sum_{i=1}^{k}(m_i - m)^2}{ks^2}}$$

We can either use Figure B.1 in the appendix of Zar (1999), obtain the power that corresponds to this phi, or use computer software. Brodgar only requires the user to type in the three means (separated by spaces), n, a, and the variance s^2 (Figure 2).

2. **The effect size or minimum detectable difference**. This is the minimum detectable difference between the mean values or the difference between the lowest and largest mean for the treatments and is denoted by (see equation 10.36 in Zar 1999):

$$d = \sqrt{\frac{2ks^2 f^2}{n}}$$

For pre-specified power, one can either look up the phi value from tables (Zar 1999), or use Brodgar to calculate automatically the minimum detectable difference (Figure 3).

3. **Estimation of required sample size.** The number of replicates used in this research that will be needed in order to achieve a desired power. The underlying formula is of the form (equation 10.34 in Zar 1999):

$$f = \sqrt{\frac{nd^2}{2ks^2}}$$

The problem is that both sides depend on n, and therefore an iterative process is required. This can either be done by hand, or by using Brodgar (Figure 4).
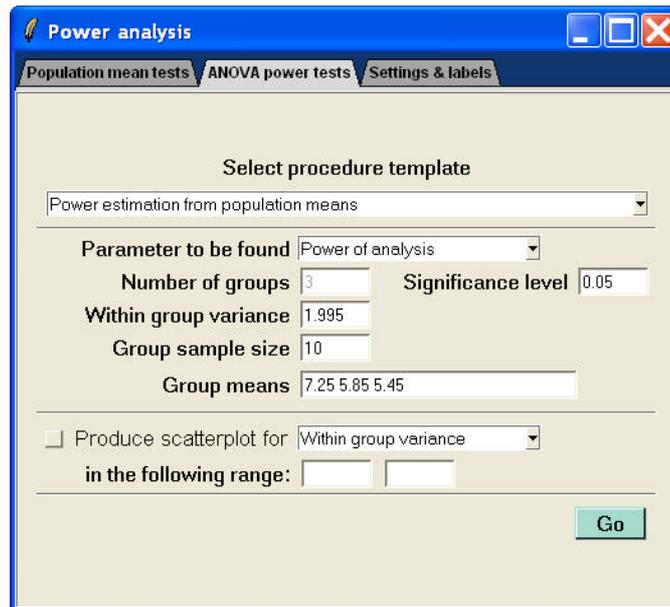
Figure 2. Estimating the power of the test. Group means must be typed in with a space as separator.
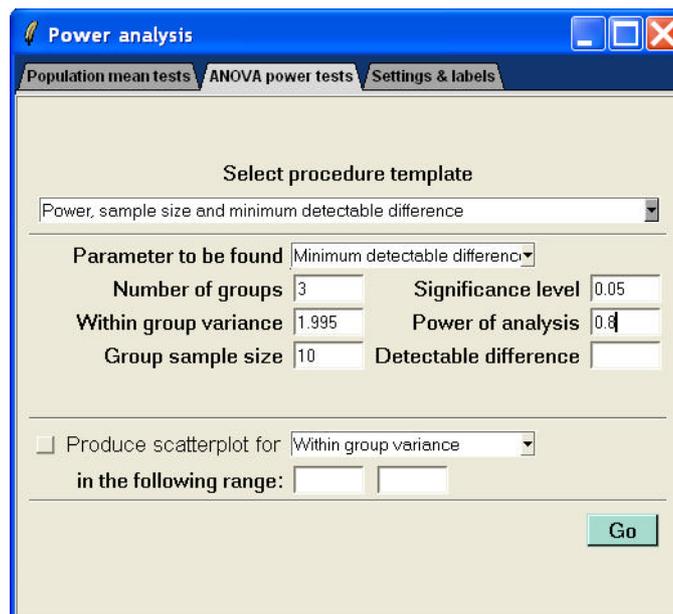

Figure 3. Estimating the minimum detectable difference for pre-specified variance, significance level, power, number of groups (k) and group size (n).

Figure 4. Estimating the required group sample size (n).

Estimating the Power of an ANOVA after is has been carried out

Once an ANOVA (one or 2-way) has been carried out, the power can be estimated using

$$f = \sqrt{\frac{(k-1)(groupMS - s^2)}{ks^2}}$$

The group MS is the MS of the explanatory variables, s^2 is the MS of the residuals and k is the number of groups. Brodgar can be used to estimate the power, see Figure 5.

Figure 5. Estimating the power of an ANOVA once it has been carried out.

2-way ANOVA and power analysis

This is explained in Section 12.7 in Zar (1999). The same formulae as above can be used except that the number of levels and replicates are adjusted.